



IIT ROORKEE



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CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS

Lecture 8: Raft Foundations-Methods of Analysis

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ERD of Foundations - Modules

1. Introduction
- 2. Shallow Foundations**
3. Pile Foundations
4. Well Foundations
5. SSI for Deep Foundations



Module - 2

Shallow Foundations



Module -2: Chapters

1. General Requirements
2. Bearing Capacity and Settlement
3. Combined Footings
4. Raft Foundations
5. Dynamic Bearing Capacity



Chapter 4: Raft or Mat Foundations

- Introduction
- Design Concept
- Codal Provisions - IS: 2950
- Determination of Young's Modulus (E_s)
- Determination of Subgrade Reaction (K)
- Rigidity of super structure and foundation
- Methods of Analysis

Introduction

Mat or raft is a **combined footing** that covers the entire area between a structure and supports all the walls and columns.

Requirement and Usage of Raft Foundations:

- When **allowable soil pressure is low**, or the **building loads are heavy**. The use of spread footings would cover more than 50% of the area and it is more economical to use mat or raft foundation.
- The **differential settlement** would be difficult to control e.g. when soil is highly compressible and erratic.
- To **reduce total settlement** above highly compressible soils by making the weight of structure and raft equal to the weight of soil excavated.

Design Concept

- Normally rafts are designed as **reinforced concrete flat slabs**.
- If the **C.G. of loads coincide with the centroid of the raft**, the upward load is regarded as a **uniform pressure** equal to the downward load divided by the area of the raft.
- The **weight of the raft is not considered** in the structural design because it is assumed to be carried out directly by the subsoil.
- Usually **reinforcement in rafts are kept heavier** than that required by the analysis which neglects the moments and shear caused by differential settlements.

Design Concept (Continue)

- The raft is divided into a **series of continuous beams** (strips) centered on the appropriate column rows.
- With the raft, now a series of beams, **the shear and moment diagrams** may be established and the mat can be designed. This will lead to a conservative design.
- The raft may also be designed as **inverted flat slab**, using heavy beams from column to column. The portion between the beam is then designed as a conventional one way or two way slab.
- A raft on the **clay may fail in shear** while the danger of shear failure on **sands** is very remote and the **settlement criteria** governs the design.

Settlements in Rafts

- A raft may undergo **large settlements** without causing harmful differential settlement. For this reason, almost **double settlement** of that permitted for footings is acceptable for raft. If a maximum settlement of 50 mm is permitted for a raft, the differential settlement is not likely to exceed 20 mm.
- **IS: 2950, Part-1: 1981** – “Code of Practice for Design and Construction of Raft Foundations, Part I: Design”, suggests that the maximum settlement should be limited to
 - (a) Raft on Clay: 65 to 100 mm
 - (b) Raft on Sand: 40 to 65 mm

Codal Provisions: IS-2950

Necessary information: Modulus of Elasticity (E_s), **Appendix A**

Field test: Plate load test (Sandy soil), IS:1888-1982

$$E_s = qB \frac{(1 - \mu^2)}{S} I_w$$

q = Intensity of contact pressure

B = Least lateral dimension of test plate

S = Settlement

μ = Poisson's ratio

I_w = Influence factor (0.82) for a square plate

Appendix-A: Determination of E_s

Effect of size of raft
$$E_s = E_p \frac{B_f}{B_p} \left(\frac{B_f - B_p}{2B_f} \right)^2$$

B_f = Width of the footing, B_p = Width of the Plate, E_p = Modulus of elasticity of plate

E_s can also be estimated from CPT: $E_s = 2C_{kd}$

where, C_{kd} = Cone resistance

E_s from laboratory test:

$$G_s = \frac{E_s}{2(1 + \mu)}$$

Appendix-B: Modulus of subgrade reaction (k)

Table-1. K for cohesionless soil (IS: 2950-1)

Soil Characteristics		K (kgf/cm ³)	
RD	SPT-N	For Dry & Moist State	For Submerged State
Loose	<10	1.5	0.9
Medium	10-30	1.5-4.7	0.9-2.9
Dense	>30	4.7-18	2.9-10.8

Appendix-B: Modulus of subgrade reaction (k)

Table-2. K for cohesive soils (IS:2950-1)

Soil Characteristic		K (kgf/cm ³)
Consistency	UCS (kgf/cm ²)	
stiff	1-2	2.7
Very stiff	2-4	2.7-5.4
Hard	4 & >4	5.4-10.8

IS : 2950 (Part I) - 1981

4. DESIGN CONSIDERATIONS

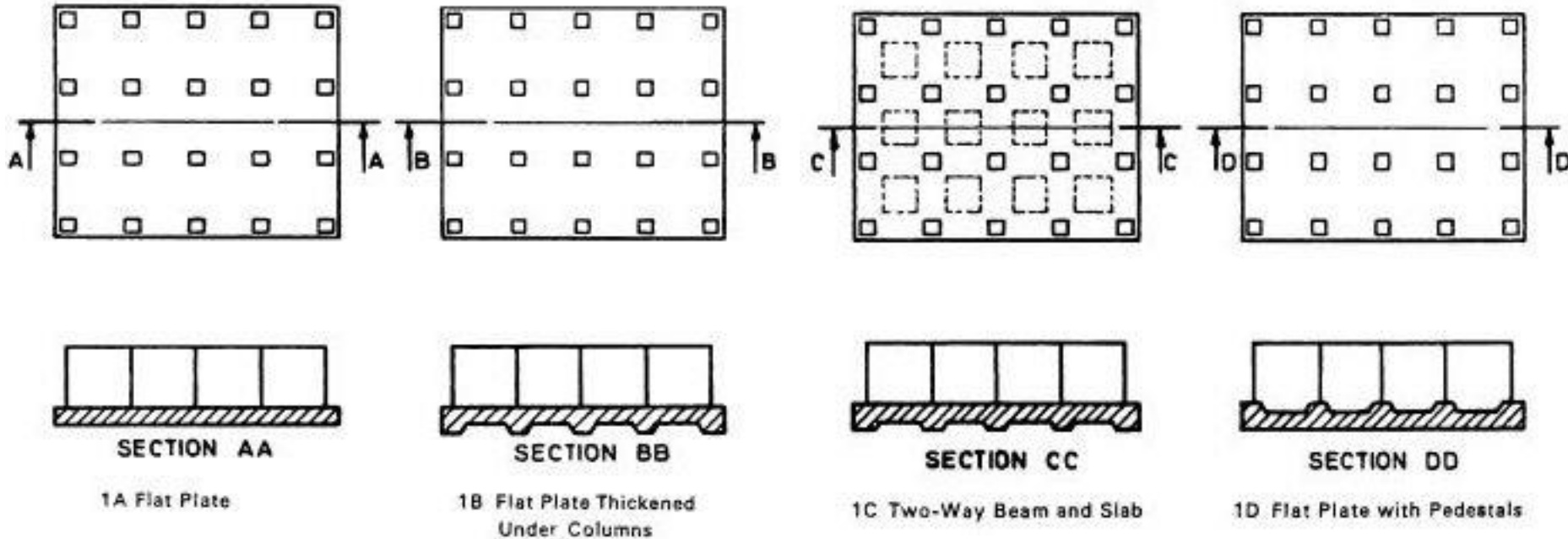
4.1 **Choice** of Raft Type: See next slide

4.2 **Allowable Bearing Pressure** - The allowable bearing pressure shall be determined in accordance with IS : 6403-1981.

4.3 **Depth of Foundation**: Shall generally be not less than 1 m

4.4 **Sub-soil Water Pressure**: The uplift due to the subsoil water shall be considered in the design.

IS : 2950 (Part I) - 1981



IS : 2950 (Part I) – 1981: Methods of Analysis

Article 5: Methods of analysis - the determination of the distribution of contact pressure

5.1 Rigid Foundation (Conventional Method) – This is based on the assumptions of linear distribution of contact pressure

5.2 Flexible Foundation - It is assumed that the subgrade consists of an infinite array of individual elastic springs

Rigid Foundations



IS : 2950 (Part I) – 1981: Rigid Foundations

5.1 Rigid Foundation (Conventional Method) - This is based on the assumptions of linear distribution of contact pressure. The basic assumptions of this method are:

- a) The foundation is **rigid relative to the supporting soil** and the compressible soil **layer is relatively shallow**.
- b) The contact pressure variation is **assumed as planar**, such that the **centroid of the contact pressure coincides** with the line of action of the resultant force of all loads acting on the foundation.

IS : 2950 (Part I) - 1981

5.1.1 This method may be used when either of the following conditions is satisfied:

- a) The structure behaves as rigid (due to the combined action of the superstructure and the foundation) with a **relative stiffness** factor $K > 0.5$ (for evaluation of K , see **Appendix C**).
- b) The **column spacing** is less than $1.75/\lambda$ (see **Appendix C**).

Appendix-C: Rigidity of structure and foundation

b = Length or breadth of the structure in bending direction

l = Spacing of the column

h_u = Length of upper column

h_l = Length of lower column

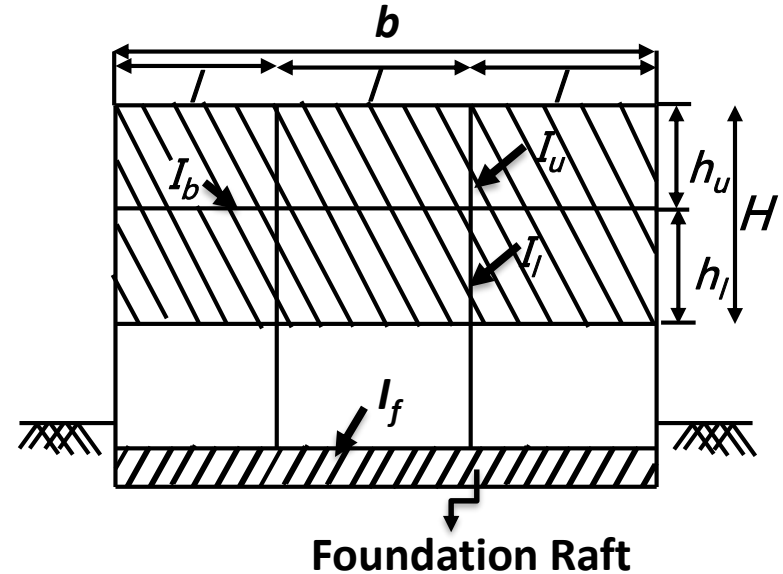
H = Total height of the infilling

I_b = Moment of the inertia of the beam

I_f = Moment of inertia of raft (foundation beam)

I_u = Moment of inertia of upper column

I_l = Moment of inertia of lower column



Appendix-C: Rigidity of structure and foundation

The flexural rigidity (EI) of the structure of any section may be estimated as

$$EI = \frac{E_l I_l b^2}{2H^2} + \sum E_2 I_b \left[1 + \frac{(I'_u + I'_l) b^2}{(I'_b + I'_u + I'_f) l^2} \right]$$

$$I'_u = \frac{I_u}{h_u}, \quad I'_b = \frac{I_b}{l}, \quad I'_l = \frac{I_l}{h_l}, \quad I'_f = \frac{I_f}{b}$$

Where

E_l = Modulus of elasticity of the infilling material

E_2 = Modulus of elasticity of the frame material

Appendix-C: Relative stiffness factor (K)

Relative stiffness factor (K): whether a structure behaves as rigid or flexible depends on the relative stiffness of the structure and the foundation of soil.

- a) For **whole structure** $K = \frac{EI}{E_s b^3 a}$
- b) For the **rectangular raft or beam** $K = \frac{E}{12E_s} \left(\frac{d}{b} \right)^3 ; \left(I = \frac{1}{12} a d^3 \right)$
- c) For **circular rafts** $K = \frac{E}{12E_s} \left(\frac{d}{2R} \right)^3 ; (b = 2R)$

Where, a = perpendicular and b =bending axis

For $K > 0.5$, the structure may be considered as rigid.

Determination of critical column spacing:

Where B is width of raft and E_c is for concrete

$$\lambda = \left(\frac{KB}{4E_c I} \right)^{\frac{1}{4}}$$

IS : 2950 (Part I) - 1981

Article 5.1.2 The raft is analyzed as a whole in each of the two perpendicular directions. The contact pressure distribution is determined by the procedure outlined in **Appendix D**.

Appendix-D: Pressure distribution (q)

The pressure distribution (q) under the raft shall be determined by conventional method:

$$q = \frac{Q}{A'} \pm \frac{Qe'_y}{I'_x} y \pm \frac{Qe'_x}{I'_y} x$$

Where,

Q = Total vertical load on raft,

A' = Total area of raft and

e'_x , e'_y , I'_x and I'_y are **eccentricities** and **moments of inertia** about the principal axes through the centroid of the section

$$I'_x = I_x - \frac{I_{xy}^2}{I_y} \quad e'_x = e_x - \frac{I_{xy}}{I_x} e_y$$

$$I'_y = I_y - \frac{I_{xy}^2}{I_x} \quad e'_y = e_y - \frac{I_{xy}}{I_y} e_x$$

I_x , I_y = **Moments of inertia** about x and y axis and e_x , e_y = **Eccentricities** in the x, y direction of the load from the centroid. I_{xy} = Double moment of area about x and y axes

Appendix-D: Pressure distribution (q)

The pressure distribution (q) under the raft shall be determined by conventional method:

$$q = \frac{Q}{A'} \pm \frac{Qe'_y}{I'_x} y \pm \frac{Qe'_x}{I'_y} x$$

For a rectangular raft, the equation simplifies to:

$$q = \frac{Q}{A} \left(1 \pm \frac{12e_y y}{b^2} \pm \frac{12e_x x}{a^2} \right)$$

Where, a and b are the dimensions of the raft in x and y directions respectively.

Note: If one or more of the values of (q) are negative, as calculated by the above formula, it indicates that the whole area of foundation is not subject to pressure and only a part of the area is in contact with the soil, and the above formula will still hold good, provided appropriate values of I_x , I_y , I_{xy} , e_x and e_y are used with respect to the area in contact with the soil instead of the whole area.

IS : 2950 (Part I) - 1981

Article 5.1.3 In cases of uniform conditions when the variations in adjacent column loads and column spacing do not exceed 20 percent of the higher value. the raft may be divided into perpendicular strips of widths equal to the distance between mid spans and each strip may be analyzed as an independent beam with known column loads and known contact pressures.

Flexible Foundations



IS : 2950 (Part I) – 1981: Flexible Foundation

Simplified Method - It is assumed that the subgrade consists of an infinite array of individual elastic springs each of which is not affected by others. The spring constant is equal to the modulus of subgrade reaction (k). The contact pressure at any point under the raft is, therefore, linearly proportional to the settlement at any point. The method may be used when the following conditions are satisfied (**Appendix E**):

- a) The **structure (combined action of superstructure and raft)** may be considered as flexible (**relative stiffness factor** $K < 0.5$)
- b) Variation in adjacent column load does not exceed 20% of higher value.

Appendix-E: Pressure distribution & moments below flexible found.

Contact pressure distribution is **assumed linear with maximum value attained under the columns and minimum at mid span**

The contact pressure for the **full width of the strip** under an interior column load at point i (p_i) can be determined as (Fig. 3A)

$$p_i = \frac{5P_i}{\bar{l}} + \frac{48M_i}{\bar{l}^2}$$

Where,

$\bar{l}$ = Average length of adjacent span (m)

P_i = Column load at point i (t), and

M_i = Moment under an interior columns located at i

Appendix-E, Continued..

The minimum contact pressure for the full width of the strip at the middle of the adjacent spans P_{ml} and P_{mr} can be determined as

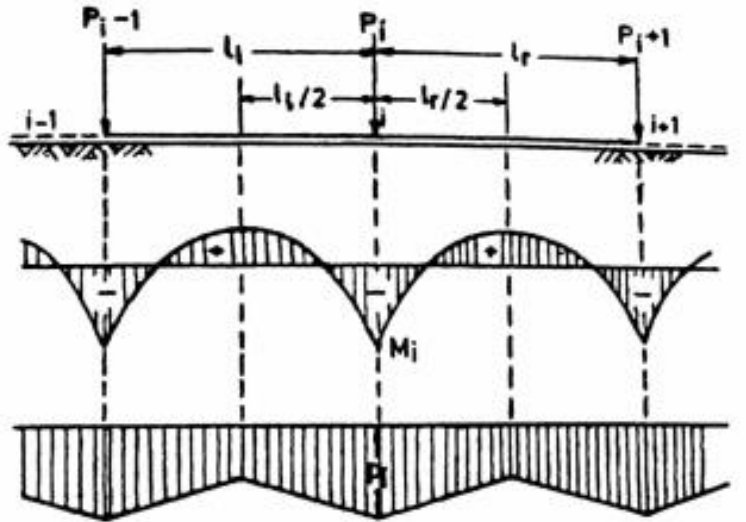
$$p_{ml} = 2P_i \frac{l_r}{l_l \bar{l}} - p_i \frac{\bar{l}}{l_l}$$

$$p_{mr} = 2P_i \frac{l_l}{l_r \bar{l}} - p_i \frac{\bar{l}}{l_r}$$

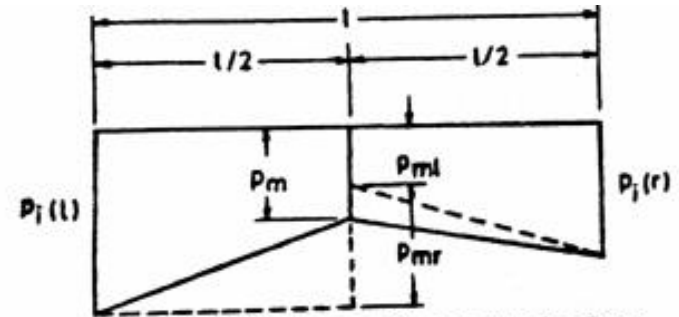
$$P_m = \frac{p_{ml} + p_{mr}}{2}$$

l_l and l_r are as shown in Fig. 3A.

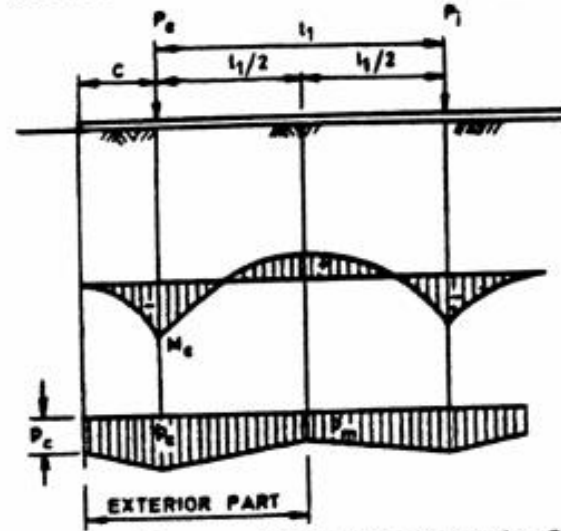
Appendix-E, Continued..



3A Moment and Pressure Distribution at Interior Column



3B Pressure Distribution Over an Interior Span



3C. Moment and Pressure Distribution at Exterior Column

Appendix-E, Continued..

If condition (a) governs the moment under the exterior columns, contact pressures under the exterior columns and at end of the strip (P_e) and at the centre (p_c), can be determined as:

$$p_e = \frac{4P_e + \frac{6M_e}{C} - p_m l_1}{C + l_1}$$

$$p_c = -\frac{3M_e}{C^2} - \frac{p_e}{2}$$

Where, P_e , p_m , M_e , l_1 and C as shown in Fig. 3C

If condition (b) governs the moment under the exterior columns, the contact pressures p_e and p_c are determined as

$$p_c = p_e = \frac{4P_e - p_m l_1}{4C + l_1}$$

Appendix-E, Continued..

The **bending moment under an interior column** located at i can be determined as:

$$M_i = -\frac{P_i}{4\lambda} (0.24\lambda\bar{l} + 0.16)$$

The **bending moment at mid-span** is obtained as

$$M_m = M_0 + M_i$$

Where,

M_0 moment of simply supported beam and others are shown in Fig.

$$M_0 = \frac{l^2}{48} [p_i(l) + 4p_m + p_i(r)]$$

Where, l , $p_i(l)$, $p_i(r)$, p_m are shown in previous slide

Appendix-E, Continued..

Conditions (a) and (b) are as follows:

The bending moment M_e , under exterior columns can be determined as the least of (Fig. 3C):

$$\text{a) } \frac{P_e}{4\lambda} (0.13\lambda l_1 + 1.06\lambda C - 0.50)$$

$$\text{b) } -\frac{(4P_e - p_m l_1)}{4C + l_1} \frac{C^2}{2}$$

IS : 2950 (Part I) - 1981

General Method – For the general case of a flexible foundation not satisfying the requirements of simplified method, this may be used. This is based on closed form solution of **elastic plate theory**. Here **plates on Winkler foundation** takes into account the restraint on deflection of a point provided by continuity of the foundation in orthogonal direction. **The distribution of deflection and contact pressure on the raft due to a column load is determined by the plate theory**. Since the effect of a column load on an elastic foundation is damped out rapidly, it is possible to determine the total effect at a point of all column loads within the zone of influence by the method of super imposition. The computation of the effect at any point may be restricted to columns of two adjoining bays in all directions. The procedure is outlined in **Appendix F**.

Appendix-F: Closed form solution of elastic plate theory

For a flexible raft foundation with non-uniform column spacing and load intensity, solution of the differential equation governing the behavior of plates on elastic foundation (Winkler Type) gives **radial moment (M_r)** **tangential moment (M_t)** and **deflection (w)** at any point by the following expressions: (per unit width, $b=1$)

$$M_r = -\frac{P}{4} \left(Z_4 \left(\frac{r}{L} \right) - (1 - \mu) \frac{Z'_3 \left(\frac{r}{L} \right)}{\left(\frac{r}{L} \right)} \right)$$

$$M_t = -\frac{P}{4} \left(\mu Z_4 \left(\frac{r}{L} \right) - (1 - \mu) \frac{Z'_3 \left(\frac{r}{L} \right)}{\left(\frac{r}{L} \right)} \right)$$

$$w = \frac{PL^2}{4D} Z_3 \left(\frac{r}{L} \right)$$

Appendix-F, Continued..

Where,

P = Column load;

r = Distance of the point under investigation from column load along radius;

L = Radius of effective stiffness; $\sqrt[4]{\frac{D}{K}}$

K = Modulus of subgrade reaction for footing of width B ;

D = Flexural rigidity of the foundation; $D = \frac{Ebt^3}{12(1-\mu^2)}$ t = thickness of raft & $b=1$

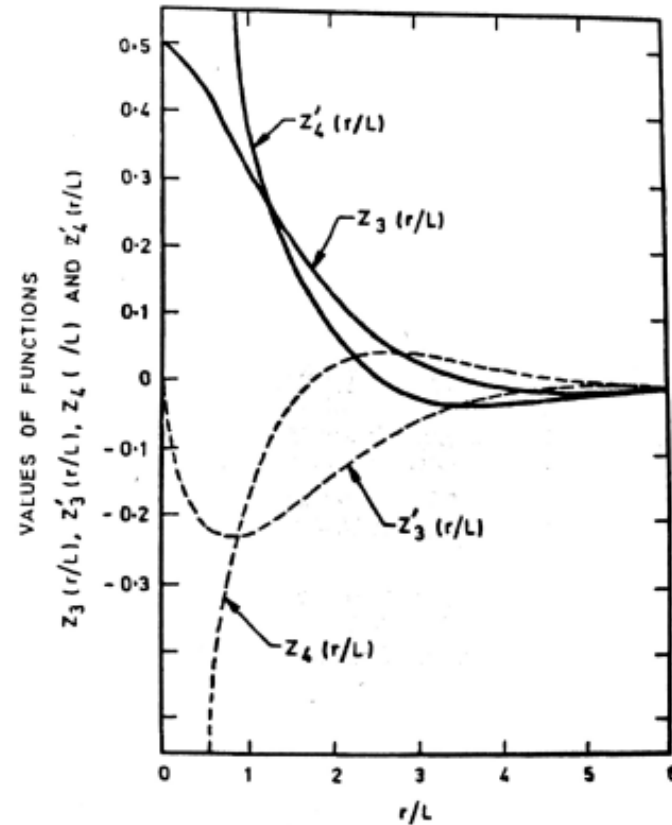
$Z_3\left(\frac{r}{L}\right)$, $Z'_3\left(\frac{r}{L}\right)$, $Z_4\left(\frac{r}{L}\right)$ are functions of shear, moment and deflection, respectively

Appendix-F, Continued..

The shear Q per unit width of raft can be determined by:

$$Q = -\frac{P}{4L} Z'_4 \left(\frac{r}{L} \right)$$

Z'_4 is a function of shear



Thank You

